

Revisiting the cosmological bias due to local gravitational redshifts

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A recent article by Wojtak *et al* (arXiv:1504.00718) pointed out that the local gravitational redshift, despite its smallness ($\sim 10^{-5}$), can have a noticeable ($\sim 1\%$) systematic effect on our cosmological parameter measurements. The authors studied a few extended cosmological models (nonflat Λ CDM, w CDM, and w_0 - w_a CDM) with a mock supernova data set. We repeat this calculation and find that the $\sim 1\%$ biases are due to strong degeneracy between cosmological parameters. When cosmic microwave background (CMB) data are added to break the degeneracy, the biases due to local gravitational redshift are negligible ($\lesssim 0.1\sigma$).

I. INTRODUCTION

The cosmological principle, that the Universe is homogeneous and isotropic on large scales, has been a successful theoretical basis for modern physical cosmology. In this idealized homogeneous and isotropic “background Universe” we can define a cosmic redshift that accounts for the redshift due to background expansion $z_c \equiv a_0/a - 1$, where a is the scale factor in the Friedmann-Lemaître-Robertson-Walker metric,

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (1)$$

and the subscript 0 denotes the quantity today. The normalization of $a(t)$ is arbitrary and chosen to be $a_0 = 1$ in this work. Natural units $c = \hbar = 1$ are used throughout the paper.

We are interested in the cosmological parameters constrained by measurements of luminosity distances. In the idealized background Universe, the luminosity distance d_L is a function of cosmic redshift z_c . Thus, the observational task is to measure the luminosity distance for objects at different redshifts and confront the data with the theoretical $d_L(z_c)$.

On cosmological scales, measuring luminosity distance is a challenging task. Ideally, for a standard candle whose luminosity is known as *a priori* knowledge, the luminosity distance can be determined with the inverse-square law for apparent brightness. In practice, the best cosmological candidate by far – the type Ia supernova – is not a perfect standard candle. The environmental dependence of supernova luminosity [1] and dust extinction [4] are accounted for by introducing some nuisance parameters in the supernova light-curve fitting methods (see [5] and the references therein for a review). Other systematics such as second-order corrections from gravitational lensing and directional averages have also been discussed in the literature [2, 6?].

Redshift by its definition is a direct observable. To linear order the observed redshift z_{obs} contains three components: $z_{\text{obs}} = z_c + z_{\text{pec}} + z_g$. The redshift due to peculiar velocity z_{pec} adds random scatter that averages out. The local gravitational redshift z_g accounts for the difference between the gravitational potentials at the points

of light reception and emission, which does not cancel if our local gravitational potential differs from the average potential in the supernova environments. If we split the supernova samples into different redshift bins and consider z_g for each subset of supernovae, z_g can be redshift dependent. Moreover, in a Universe with time-dependent gravitational potentials, z_g also contains an integrated Sachs-Wolfe contribution that has redshift dependence. Simulations show, however, that z_g can be approximately treated as a constant at $z \lesssim 1$ and its amplitude is $\leq 4 \times 10^{-5}$ at 95% confidence level (C.L.) [7]. This seemingly tiny systematic error by far has been ignored in supernova data analysis. Ref. [7] for the first time computed the impact of a nonzero z_g and found $\sim 1\%$ bias on the best-fit cosmological parameters. For modern precision cosmology, 1% bias is a non-negligible effect. However, for the extended models (nonflat Λ CDM, w CDM, and w_0 - w_a CDM) studied in Ref. [7], cosmological parameters are strongly degenerate with current supernova data *alone*. The 1% shift of best-fit parameters in the flat degeneracy direction may not be a significant effect. The purpose of this paper is to demonstrate that the systematic errors due to z_g becomes negligible when we use cosmic microwave background (CMB) data to break the degeneracy between parameters.

II. SYSTEMATIC ERRORS WITH SUPERNOVA DATA ALONE

The standard treatment of supernova likelihood is to absorb the random scatter z_{pec} into an additional uncertainty of luminosity distance. The local gravitational redshift is a biased error that, in principle, cannot be treated in the same way. With random scatter ignored, a nonzero z_g gives an observed redshift

$$z_{\text{obs}} = z_c + z_g, \quad (2)$$

and an observed luminosity distance

$$d_{L,\text{obs}} = d_L(z_c)(1 + z_{\text{obs}})/(1 + z_c), \quad (3)$$

where $d_L(z_c)$ is the theoretical luminosity distance function. (See Eq. (3.3) of Ref. [7].) The extra factor $(1 + z_{\text{obs}})/(1 + z_c)$, which has been ignored in Ref. [7],

accounts for the fact that a local gravitational redshift also changes the apparent brightness via an additional redshift of photons and an additional time dilation. We find that, however, for all cases discussed in this paper the $(1 + z_{\text{obs}})/(1 + z_c)$ correction does not lead to a noticeable difference.

Following the approach in Ref. [7], we use the redshifts (treated as z_c) and errors in the apparent magnitude of 580 Union2.1 SN samples [8]. Mock data with a fiducial Λ CDM cosmology ($\Omega_m = 0.3$) and a fiducial z_g are produced with Eqs. (2)-(3). For mock data sets with $z_g = 0, \pm 4 \times 10^{-5}$, we compute the best-fit parameters, respectively. The 68.3% C.L. and 95.4% C.L. constraints on parameters are computed via Monte Carlo Markov chain (MCMC) simulations.

For illustration purposes, we consider a nonflat Λ CDM model and a flat w CDM model. (See [7] for the definitions of these models.) For supernova data the Hubble constant H_0 is degenerate with the supernova absolute luminosity and does not affect the likelihood. The likelihood depends on, in the nonflat Λ CDM case, Ω_k (curvature of the Universe) and Ω_m (fractional energy of matter), and in the w CDM case, w (dark energy equation of state) and Ω_m . The dark energy fractional energy Ω_Λ is constrained by $\Omega_k + \Omega_m + \Omega_\Lambda = 1$ ($\Omega_k = 0$ in w CDM case).

As shown in Fig 1, we find that the systematic errors on cosmological parameters agree with Ref. [7]. However, the $\sim 1\%$ bias of best-fit parameters are aligned with the degeneracy direction where the posterior likelihood is flat. Compared to the 1σ (68.3% C.L.) constraint, the systematic errors are small.

III. SYSTEMATIC ERRORS WITH SUPERNOVA + CMB

In this section we combine the publicly available Planck CMB likelihood [9] and the Union2.1 supernova likelihood to constrain cosmological models, with different fiducial z_g assumed. The Planck likelihood includes CamSpec temperature power spectrum likelihood (2013 release) and a low- ℓ WMAP polarization constraint. The supernova likelihood code is modified to include the z_g correction.

We run Monte Carlo Markov chains with the publicly available code COSMOMC [10], with the standard settings, namely, flat priors on six parameters $\Omega_b h^2$ (baryon density), $\Omega_c h^2$ (cold dark matter density), θ (angular extension of sound horizon on the last scattering surface), τ (CMB optical depth), $\ln A_s$ and n_s (logarithmic amplitude and index of the primordial power spectrum of curvature fluctuations). For nonflat Λ CDM model and w CDM model we add Ω_k and w with a flat prior, respectively.

As shown in Fig. 2, the impact of z_g becomes invisible with the addition of CMB data. In Table I and Table II we compare the mean and standard deviations of param-

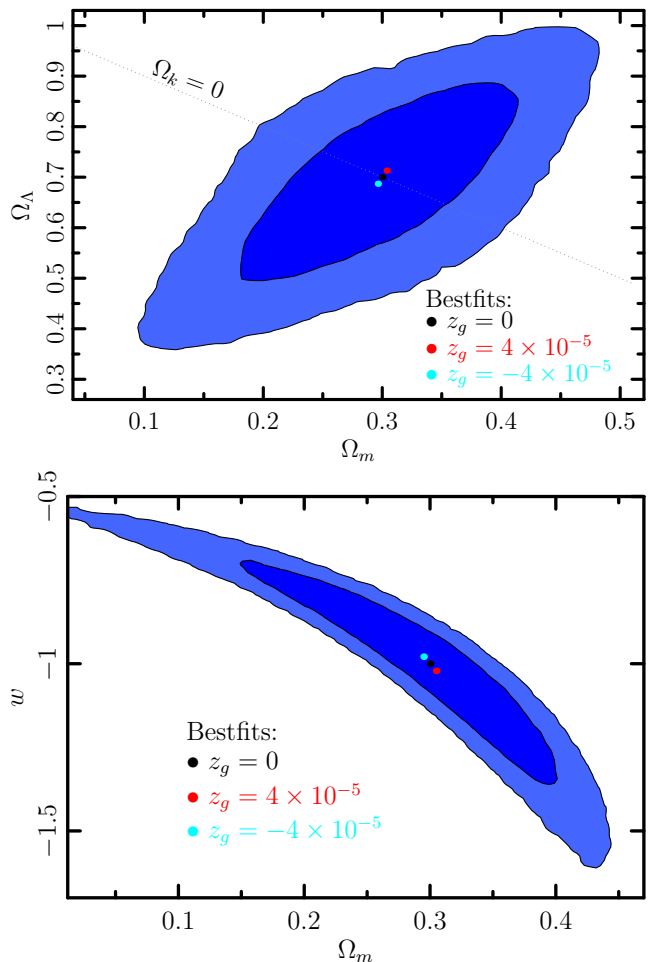


FIG. 1. The systematic errors due to a local gravitational redshift. Upper panel: nonflat Λ CDM model; lower panel: flat w CDM model. The contours are 68.3% C.L. and 95.4% C.L. constraints for the $z_g = 0$ case.

TABLE I. Nonflat Λ CDM; CMB + SN

| | $z_g = 0$ | $z_g = -4 \times 10^{-5}$ | $z_g = 4 \times 10^{-5}$ |
|------------------|--------------------|---------------------------|--------------------------|
| Ω_m | 0.344 ± 0.042 | 0.344 ± 0.043 | 0.345 ± 0.042 |
| Ω_Λ | 0.665 ± 0.033 | 0.665 ± 0.032 | 0.664 ± 0.032 |
| Ω_k | -0.009 ± 0.011 | -0.009 ± 0.011 | -0.009 ± 0.011 |

eters and find that in all cases the biases due to a nonzero z_g are less than 0.1σ .

IV. CONCLUSIONS AND DISCUSSIONS

The findings of Ref. [7], that a local gravitational redshift can cause $\sim 1\%$ systematic errors, raise a question whether or not we should include the z_g correction in the supernova data analysis. In this paper we have demonstrated that, for current supernova data, as long as CMB data are included to break the parameter degeneracy, the

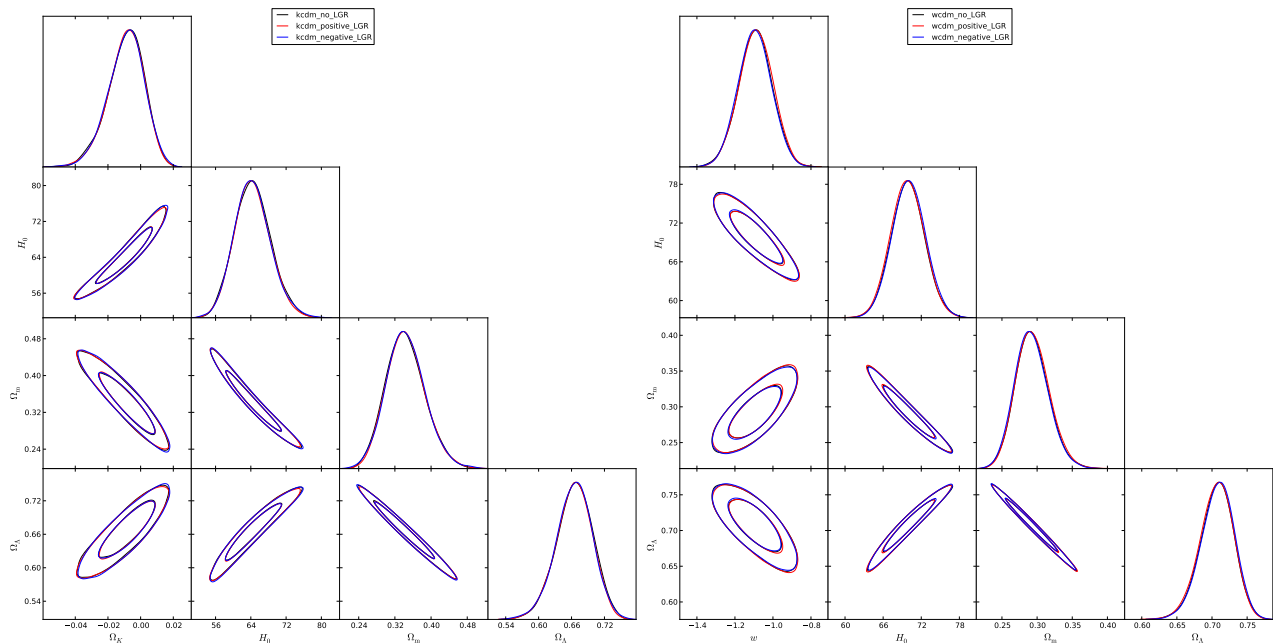


FIG. 2. The 68.3% C.L. and 95.4% C.L. constraints on nonflat Λ CDM cosmology (left panel) and w CDM cosmology (right panel). The red, blue and black lines are for $z_g = 4 \times 10^{-5}$, $z_g = 0$, and $z_g = -4 \times 10^{-5}$ respectively. Data sets: Planck 2013 CamSpec + WMAP low- ℓ polarization + Union2.1 Supernova.

TABLE II. Flat w CDM; CMB + SN

| | $z_g = 0$ | $z_g = -4 \times 10^{-5}$ | $z_g = 4 \times 10^{-5}$ |
|------------|--------------------|---------------------------|--------------------------|
| Ω_m | 0.293 ± 0.023 | 0.292 ± 0.023 | 0.294 ± 0.023 |
| w | -1.093 ± 0.087 | -1.096 ± 0.086 | -1.089 ± 0.087 |

impact of the local gravitational redshift is negligible. Thus, no modification of current supernova likelihood is on demand.

In Sec. II and Sec. III we have used different sets of parameters. In particular, in the supernova-data-alone case we have used a flat prior on Ω_m , and for CMB + SN we have used flat priors on $\Omega_b h^2$, $\Omega_c h^2$ and θ . (Ω_m is a derived parameter with a complicated prior.) To make a

more direct comparison, we replace the CMB likelihood with a “compressed CMB likelihood” that contains only the distance information, that is, a Gaussian constraint $R = \sqrt{\Omega_m H_0 d_A(z_*)} = 1.7488 \pm 0.0074$ [11], where d_A is the comoving angular diameter distance and $z_* = 1089$ is the redshift at recombination. The advantage of using this compressed likelihood is that R can be written as a function of Ω_m , Ω_k and w . For supernova and this compressed CMB likelihood, we use the same parametrization and priors as in Sec. II and find that the systematic errors due to z_g remain negligible ($\lesssim 0.1\sigma$).

Whether a systematic error can be ignored depends on the precision of observations. Local gravitational redshift may still be a potential source of information contamination for future supernova observations. We leave the forecast of forthcoming supernova data to future work.

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